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Numerical Simulation of Cylindrical Laminated Shells Under Impulsive Lateral Pressure

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The finite element approach used to investigate the behavior of cylindrical laminated shells when acted upon by sudden dynamic loads is described. When solving the dynamic buckling problem, two major time-integration techniques are used, namely, the implicit integration operator in its Newmark method implementation and the explicit central difference scheme. Their performance and efficiency are compared, and the conclusion is made that, unlike some other types of dynamic problems for dynamic shell stability, the explicit finite element approach shows good accuracy of results combined with an excellent computational efficiency. Therefore, this approach is applied in the present investigation. The loading acting upon the shell structure consists of lateral suddenly applied fluid pressure, which remains normal to the shell surface as the shell deforms. Two different dynamic buckling criteria are used and combined with the finite element method in the proposed solution procedure. A new approach for assessing shell stability by using the variation of the volume enclosed by the shell is proposed. A demonstration example is presented. The performance of the present approach is compared with previously published results.

I. Introduction

CYLINDRICAL shells have been used extensively in all types of structures. They are subjected to various loadings, very often dynamic in nature. The lateral pressure loading, which is a common case, may cause deformations of unacceptably large amplitudes and could lead to loss of stability and collapse of the structure. Because of their numerous advantages, composite materials are increasingly used in shell structure design. Therefore, the problems of investigating the behavior of laminated cylindrical shells subjected to different static and dynamic loadings have drawn considerable attention of scientists, researchers, and designers since the 1950s. In the reviews of Svalbonas and Kalnins,[1] Hsu,[2] and Simitses,[3] different aspects and phenomena related to the dynamic stability of cylindrical shells have been included. The problem of dynamic buckling of axially loaded shells was first attempted by Volmir,[4] who used Galerkin's method. Coppa and Nash[5] and Roth and Klosner[6] applied the potential energy method to study this problem. Tamura and Babcock[7] investigated the dynamic buckling of cylindrical shells with geometric imperfections, applying the Budiansky–Roth criterion.[8] The dynamic stability of suddenly loaded laminated cylindrical shells and the effect of static preloading on the dynamic critical load was studied by Simitses.[9,10] Most of the preceding works investigated the shell structure behavior at axial loading (compression). Huyan and Simitses[11] considered the problem of dynamic buckling of geometrically imperfect cylindrical shells under bending moments, and Shaw et al.[12] did so for the problem under torsional loading. There are also some works regarding the dynamic buckling of cylindrical shells subjected to lateral pressure: Al-Hassani et al.[13] presented and compared theoretical and experimental results for thin-walled metal tubes under a pressure pulse. Mustafa et al.[14] studied the dynamic buckling response of tubes immersed in water and subjected to external pressure. Schokker et al.[15] investigated the dynamic instability of interior-ring stiffened composite shells under hydrostatic pressure, and Gu et al.[16] studied the dynamic plastic buckling of cylindrical shells under general external impulsive loading using the energy criterion.

The present study is aimed at determining the critical suddenly applied lateral pressure for cylindrical laminated shells with and without statically applied preloading. The solution approach and the results acquired with it can be applied in solving problems that occur in different real-world applications: submarines subjected to underwater explosion, airplanes or fuel tanks under gust loading, submerged pipelines under impact loading, and jet engine casings under rotor imbalance, to name just a few. The contribution to the state of the art is summarized by the following two points:

1) Comparison of explicit vs implicit finite element solutions for such problems.

2) In determining the instability of shells under suddenly applied load, the buckling criteria require knowledge of points on the shell where maximum displacements are observed. The present study uses the shell volume instead of displacements to assess buckling. This is more convenient because there is no necessity to know where the maximum displacements will be.

II. Statement of the Problem

A cylindrical laminated shell (Fig. 1a) of length L, radius R, and total wall thickness h_t is subjected to uniformly distributed lateral pressure p, which is suddenly applied. The solutions to two basic problems are sought: First, the critical pressure p_{cr}, which causes buckling of the shell, is defined for different loading time durations starting from a very small time duration to practically infinity without static preloading (Fig. 1b); second, in the presence of previously applied static preloading, values of p_{cr} for infinite duration are sought for different values of the statically applied preloading (Fig. 1c). The problems are approached numerically using the finite element method. The investigated shells are geometrically imperfect. The problems just described can be solved for different shell geometries (L/R and R/h_t ratios), different lamina stacking sequences, and different imperfections. Thus the relevant response relationships can be acquired. Furthermore, this approach can be extended for nonuniform and time-dependent loading.

Because buckling is usually associated with significant displacements, the loading applied should follow the geometry and always be normal to the shell surface to enable the accurate modeling of the actual pressure loading. Because the formulation of the finite element code used is fully geometrically nonlinear, two types of loading can be used for that purpose: pressure loading applied upon the element surfaces and hydrostatic fluid loading applied by filling in or surrounding the cylinder with fluid and prescribing negative

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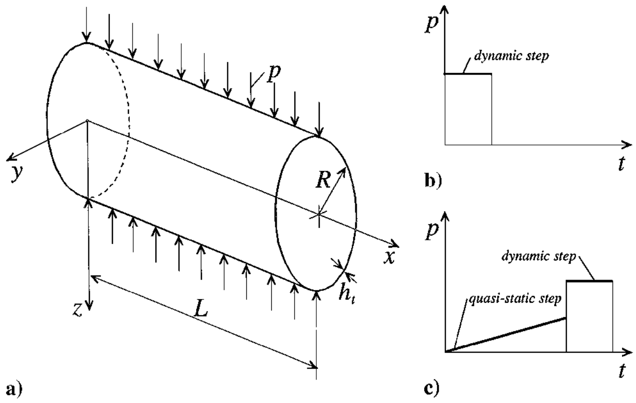

Fig. 1 Model geometry and loading.

or positive pressure change of the fluid. Thus, the pressure applied follows the shell as the geometry changes and is always directed along the normal to the shell surface. This type of loading produces more realistic results, as opposed to the radial pressure loading.[17]

To trigger buckling, initial imperfections are introduced in the geometry of the model. A linear stability analysis on the geometrically perfect structure is performed to establish probable collapse modes. The imperfection is then introduced by adding the scaled modes to the geometry of the perfect structure. A geometrically nonlinear analysis is then performed on this imperfect structure. The imperfections are introduced as follows: An eigenvalue problem is numerically solved with the perfect geometry. Resulting from this analysis, normalized eigenmode displacements along the two transverse axes are output. To produce the imperfect shape, these displacements are scaled, summed, and then added to the corresponding coordinates of the perfect geometry. Only the first three eigenmodes are used here to produce the initial geometric imperfections. Because of the impulsive nature of the loading, higher-frequency modes will be excited as well, and the buckled shape will probably be different from the shape produced by the study. However, the critical pressure, which is of main interest here, will not change significantly. Our decision to use only the first three eigenmodes in producing the imperfect shape is based on analysis of the geometry in hand, involving more than three (up to nine) mode shapes, which yielded basically the same results for the critical pressure.

III. Theoretical Formulation

A. Equations of Equilibrium

The equations of equilibrium governing the dynamic response of the system are

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{R^{\text{ext}}(t)\} \tag{1}$$

where $[M]$ is the mass matrix; $[C]$ is the damping matrix; $[K]$ is the stiffness matrix; $\{\ddot{u}\}$, $\{\dot{u}\}$, and $\{u\}$ are the nodal accelerations, velocities, and displacements vectors, respectively; and $\{R^{\text{ext}}(t)\}$ is the external forces vector. Equations (1) can be rewritten in the form

$$\{F_I(t)\} + \{F_D(t)\} + \{R^{\text{int}}(t)\} = \{R^{\text{ext}}(t)\} \tag{2}$$

where $\{F_I(t)\} = [M]\{\ddot{u}\}$, $\{F_D(t)\} = [C]\{\dot{u}\}$, and $\{R^{\text{int}}(t)\} = [K]\{u\}$ are the inertia force, the damping force, and the internal forces vectors, all of which are time dependent.

Equations (1), which represent a system of nonlinear second-order differential equations, are solved with the help of the ABAQUS (Ref. 18) finite element package using direct time-integration techniques.

B. Direct Time Integration

When solving dynamic problems with the finite element method, the solution is sought by dividing the total response time of the system into much smaller time intervals called *time steps* or *time increments.* The equilibrium equations are solved, and the values of the unknowns are determined at time $t + \Delta t$ based on knowledge of their values at time t (quasilinearization). Using these values, we continue solving the linearized differential equations at $t + 2\Delta t$ and so on for the entire response time of the system. When employing time integration, ABAQUS uses two basic methods called the *implicit and explicit integration operators.*

The implicit integration operator definition is completed by the Newmark[19] formulas for displacement and velocity integration:

$$\begin{aligned} u|_{t+\Delta t} &= u|_t + \Delta t\dot{u}|_t + \Delta t^2[(1/2 - \beta)\ddot{u}|_t + \beta\ddot{u}|_{t+\Delta t}] \\ \dot{u}|_{t+\Delta t} &= \dot{u}|_t + \Delta t[(1-\gamma)\ddot{u}|_t + \gamma\ddot{u}|_{t+\Delta t}] \end{aligned} \tag{3}$$

where β and γ are parameters of the system. Thus, expressing the velocities and accelerations at $t + \Delta t$ in terms of the displacements at $t + \Delta t$ and substitution into Eqs. (1) yields

$$[\bar{K}]\{u|_{t+\Delta t}\} = \{\bar{F}|_{t+\Delta t}\} \tag{4}$$

where $[\bar{K}] = [\bar{K}([K], [M], [C], \Delta t)]$ is the effective stiffness matrix and

$$\{\bar{F}|_{t+\Delta t}\} = \{\bar{F}(\{F|_{t+\Delta t}\}, [M], [C], \Delta t, \{u|_t\}, \{\dot{u}|_t\}, \{\ddot{u}|_t\})\}$$

is the effective load at time $t + \Delta t$.

The explicit dynamic analysis in ABAQUS is based on integrating the equations of motion for the system using the explicit central difference formula:

$$\begin{aligned} \ddot{u}|_t &= (1/\Delta t^2)(u|_{t-\Delta t} - 2u|_t + u|_{t+\Delta t}) \\ \dot{u}|_t &= (1/2\Delta t)(u|_{t+\Delta t} - u|_{t-\Delta t}) \end{aligned} \tag{5}$$

Thus Eqs. (1) take the form

$$[\bar{M}]\{u|_{t+\Delta t/2}\} = \{\bar{F}|_t\} \tag{6}$$

where $[\bar{M}] = [\bar{M}([M], [C], \Delta t)]$ is the effective mass matrix and

$$\{\bar{F}|_t\} = \{\bar{F}(\{F|_t\}, [K], [M], [C], \Delta t, \{u|_t\}, \{u|_{t-\Delta t/2}\})\}$$

is the effective load at time t.

Equations (4) are solved at time $t + \Delta t$, and the corresponding solution methods are called *implicit*, whereas Eqs. (6) are solved at time t and the corresponding methods are called *explicit.* The explicit central difference operator is very convenient when a lumped mass matrix can be assumed and velocity-dependent damping can be neglected. Then $[\bar{M}]$ is a diagonal matrix, and the solution is achieved automatically without having to solve the system of equations. On the other hand, the explicit operator is only conditionally stable, which means that to obtain accurate results the time step must be smaller than a certain critical value that is defined by the mass and stiffness properties of the complete element assemblage. Depending on the particular problem, the stable time increment of the model may be very small, which would require too many steps to solve and therefore would require an extremely large CPU time. In such cases the implicit method, which is unconditionally stable, may provide better efficiency. In our case, to access the performance of both time integration operators, shells with different characteristics and loading values below and above the critical buckling pressure were investigated using both the implicit (implemented in ABAQUS/Standard) and the explicit (implemented in ABAQUS/Explicit) methods. The results showed that both operators predict basically the same behavior, but in all cases the explicit method required only a fraction of the CPU time used by the implicit scheme. Thus, the conclusion was drawn that the explicit analysis can be used very efficiently to study the dynamic buckling behavior of cylindrical laminated shells, and thereafter all investigations were performed with ABAQUS/Explicit.

C. Dynamic Buckling Criteria

The concepts and methodologies used to estimate critical conditions for suddenly loaded elastic systems have been classified into three basic groups.[10]

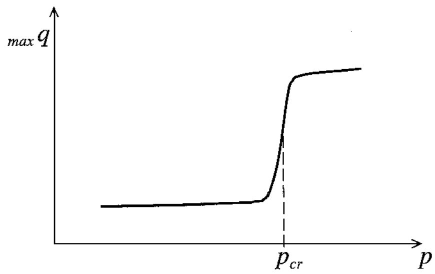

Fig. 2 Determining the critical loading by the Budiansky–Roth criterion.[8]

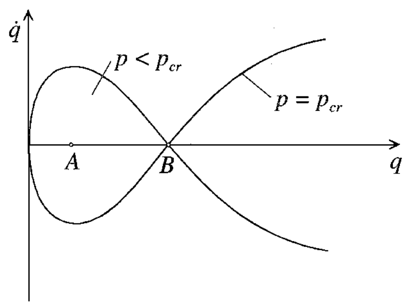

Fig. 3 Determining the critical loading by the phase-plane criterion.

Equation of Motion Approach: Budiansky–Roth[8] Criterion

The equations of motion are solved for various values of the loading, and the value at which there is a significant jump in the response is assumed critical. When monitoring the system response through displacements of selected points for small values of the loading parameter small oscillations are observed, the amplitudes of which gradually increase as the loading is increased. When the loading reaches its critical value, the maximum amplitude experiences a large jump (Fig. 2). Therefore, implementation of this criterion requires solving the equations of motion for different values of the loading parameter and then plotting the displacement amplitude vs loading curve from which the critical loading value is determined.

Phase-Plane Approach

If q is a parameter used to monitor the system response (usually a typical displacement), the phase plane is the plane in which the phase trajectories (plots of $\dot{q} = \mathrm{d}q/\mathrm{d}t$ vs q) lie (Fig. 3). For loads smaller than the critical, the system simply oscillates about the static equilibrium point A and, at loadings equal to or greater than the critical escaping motion, indicates that buckling occurs through the unstable static equilibrium point B.

Total Potential Energy Approach

In this approach critical conditions are related to characteristics of the system total potential and are applicable to conservative systems only.

Most numerical studies on dynamic buckling of laminated shells apply the Budiansky–Roth criterion[8] to determine the critical loading. The present study applies both this criterion and the phase-plane criterion. All results show that for the problems in hand both criteria predict equal values for the critical loading within reasonable accuracy.

To assess stability, the volume change of the shell cylinder is used in this study. When using displacements with buckling criteria, the points where the displacements are to be traced have to be carefully selected. Otherwise, the results produced may be meaningless and inaccurate. Furthermore, when changing some parameters of the system, the buckling mode changes, which means that the displacements of different points should be traced to investigate the response. The volume change, which can be regarded as an overall estimate of the displacements, in our opinion is more convenient when assessing stability.

IV. Demonstration Example

Based on the preceding results, the explicit finite element package ABAQUS/Explicit was used to perform a dynamic buckling analysis of the model shown on Fig. 1. Both Budiansky–Roth[8] and phase-plane criteria were used to evaluate the dynamic critical pressure. The investigated model consists of a composite cylinder loaded with suddenly applied lateral pressure. The cylinder has radius $R = 17.78$ cm (7 in.) and length $L = 35.56$ cm (14 in.); the shell has total thickness $h_t = 0.3556$ cm (0.14 in.) and consists of a total of 20 layers with the lamina-stacking sequence $[90\ \mathrm{deg}_5/0\ \mathrm{deg}_5]_S$. Both cylinder ends are simply supported. All laminae are made of the same homogeneous elastic material. The loading is applied as the negative hydrostatic fluid pressure of a fluid, which fills the cylinder. The pressure is suddenly applied at the beginning and is kept constant throughout the analysis. For a more detailed description of the model, the reader is referred to Ref. 20, where there are also results from varying different model parameters and their influence over the buckling behavior. The effect of static preloading over the shell buckling is also investigated there.

Because of the symmetry, only half of the shell along its axis is discretized, prescribing symmetric boundary conditions at the midsection plane: no axial displacements and no rotations around the radial axis and the circumference tangent. The finite element mesh has 48 elements around the shell circumference and 8 elements along the cylinder length. S4R first-order shear deformation shell elements are used.[18]

V. Results and Discussion

A. Validation of Solution Methodology

To assess the performance of the methodology, some of the models investigated in Ref. 15 are used. Validation is achieved by investigating the dynamic response of the laminated shells considered in Ref. 15. Results for the dynamic critical loading reported in Ref. 15 by Schokker et al. and results obtained with the present methodology, considering the same models, are presented in Table 1. The values for the dynamic critical pressure acquired by the two different methodologies show good agreement, the small differences probably being because of the different imperfection shapes used.

B. Results and Discussion

Illustrative results from the investigation of the demonstration example are presented in Figs. 4–11. As is shown in Fig. 4, when the critical pressure value for a small increase in the applied lateral pressure is reached, we witness an abrupt change in the system response. The phase-plane curves (Figs. 5 and 6) also indicate that the critical loading has been reached. Several more iterations on the applied pressure with values below and above the critical provide the information for the maximum amplitude-vs-pressure curve (Fig. 7) from which the same value for the critical pressure is derived.

Note that to be able to get good results from the analyses the points for which displacements are to be monitored are to be carefully chosen; otherwise, the plots produced may be rather obscure and confusing. In our case we monitored the radial displacements of several points from the midsection of the cylinder where the largest displacement values were expected. Examples with randomly chosen points proved that the proper choice is essential for the results to be adequate. Furthermore, because we are dealing with a closed-volume structure, we can use the volume to monitor its response. Because the volume change is related to the displacements of all nodes, it can be used as an overall estimate for the shell behavior instead of the nodal displacements.

Table 1 Dynamic critical pressure

L/R	q_{cr} [MPa], after Ref. 15[a]	q_{cr} [MPa], from present study
2	0.97	0.95
4	0.64	0.67
6	0.48	0.51
8	0.45	0.46

[a]Results are from Ref. 15, p. 51, Table 16.

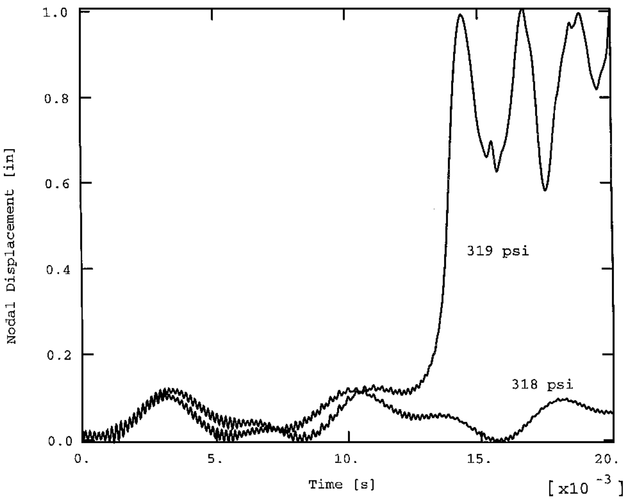

Fig. 4 Displacement curves for loading values immediately below and equal to the critical.

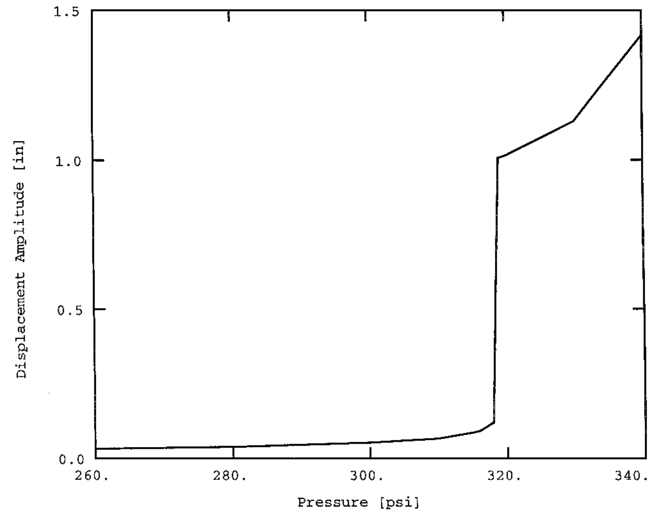

Fig. 7 Budiansky–Roth criterion[8] plot using displacement amplitudes.

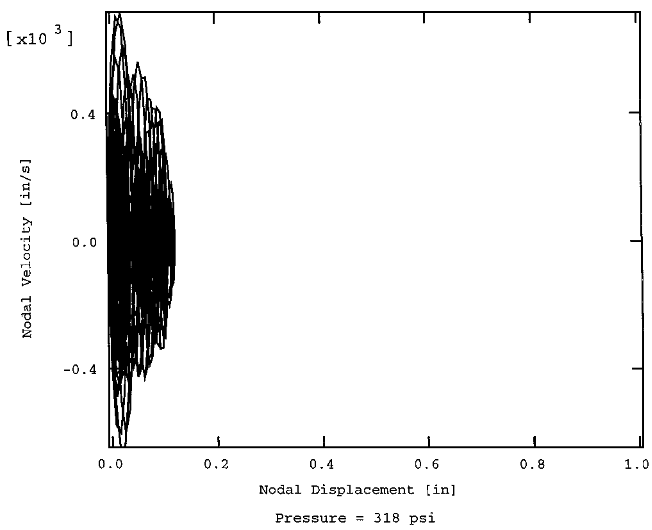

Fig. 5 Phase trajectory for loading value immediately below the critical.

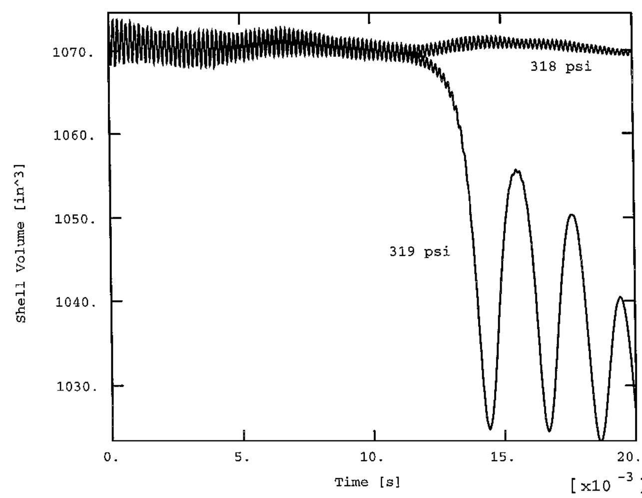

Fig. 8 Volume change curves used to assess the shell response.

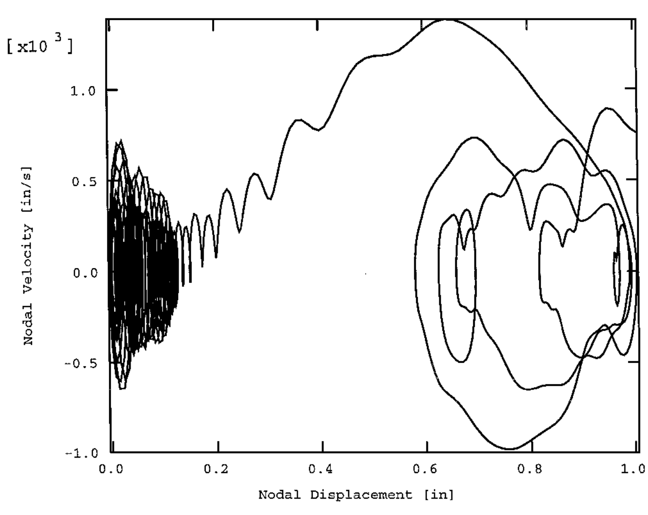

Fig. 6 Phase trajectory for loading value equal to the critical.

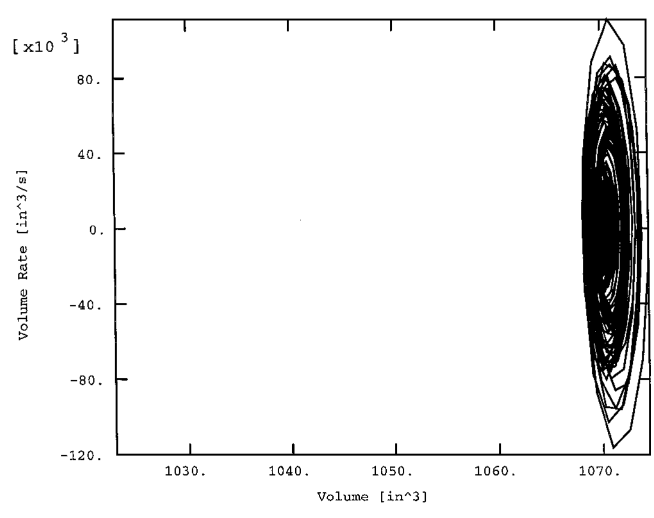

Fig. 9 Phase trajectory for loading value immediately below the critical, using the shell volume to assess its response.

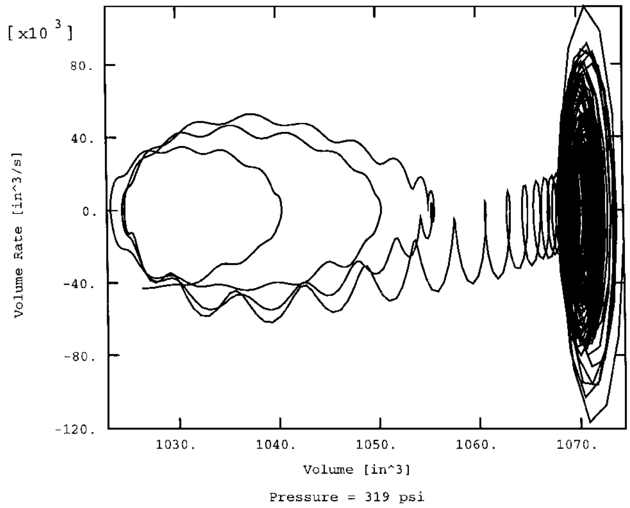

Fig. 10 Phase trajectory for loading value equal to the critical, using the shell volume.

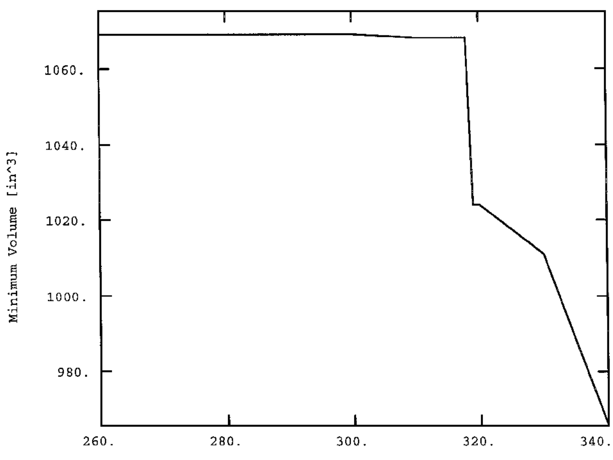

Fig. 11 Budiansky–Roth criterion[8] plot using the shell volume.

As is shown in Figs. 8–11, the shell volume can be successfully used to estimate the dynamic buckling pressure. To be able to plot the phase-plane curve in this case, the volume curve is differentiated with respect to time to get the volume rate and plot it vs the volume.

VI. Conclusions

The present study compares the two most widely used finite element time-integration techniques for the solution of dynamic problems: the implicit and the explicit integration. The study demonstrates that when investigating the dynamic buckling of cylindrical laminated shells the explicit method is very attractive, combining both good accuracy and excellent computational efficiency. The study also implemented two different dynamic buckling criteria and compared their performance and results. A new approach for assessing shell stability by using the variation of the volume enclosed by the shell is described and is suitable for investigating the stability of shells with closed volumes or such that can be incorporated in a closed-volume shape. The solution approach herein presented for investigating the dynamic buckling of laminated shells proved to be very efficient in solving many problems with different input data and in investigating the shell response sensitivity to different system parameter variations.

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